Modified Collapsed Dimension Method for Radiative Heat Transfer Problems

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Introduction

COLLAPSED dimension method (CDM) is one of the ray tracing methods used for the solution of radiative transfer problems. Development of this method is partly based on the works of Shih and Chen[1] and Shih and Ren[2] on the discretized intensity method. Some light on the method was thrown by Blank.[3] Subsequent development of the method was made by Blank and Mishra.[4] Detailed description of this method is available in Ref. 5.

In CDM, three-dimensional radiative information is mapped into a two-dimensional solution plane in terms of effective intensity (EI) and optical thickness coefficient (OTC). Thus, unlike other methods, analysis and computations in this method are performed in a two-dimensional solution plane instead of three-dimensional space.

CDM has been used for the solution of radiative transfer problems with high accuracy.[4,5] This method has been found to work for a wide range of optical thickness (very low to high optical thickness).[5] Furthermore, CDM has also been found to work well for the conjugate mode heat transfer problems.[6,7]

In CDM, for determination of heat flux and temperature information, at each point of interest EIs have to be integrated over planar angle in the solution plane. These angular integrations are performed by dividing the planar angle into intervals of equal sizes. In each subinterval, EI is assumed isotropic. Hence, for higher accuracy, CDM requires more EIs and, thus, higher computational time.

In the present work, with the objective of making CDM more economical, the method is modified. In the modified CDM (MCDM), angular integrations of the EIs are performed differently. The discretization and integrations performed in this method originate from the concept of the discrete ordinate method (DOM). Here, the planar angle is divided into a finite number of subintervals according to the number of Gaussian quadrature points considered. Hence, in MCDM, angular subintervals are unequal and instead of considering average values of interval, the weighted mean corresponding to the Gauss points is considered. With this, the method becomes more realistic, and it derives computational efficiency with a more realistic representation of the radiative transfer process.

In the present work, improvements in MCDM over CDM are tested by solving radiative transfer problems in one- and two-dimensional Cartesian enclosures with gray and homogeneous, absorbing, emitting, and anisotropically scattering medium. Both radiative and nonradiative equilibrium situations are considered. MCDM and CDM results are compared with results from the exact method[5] and DOM.[8]

Formulation

The distribution of EIs and angular thickness of the discrete planes (DP) in both CDM and MCDM are shown in Figs. 1a and 1b. As shown in Fig. 1a, the distribution of EI for CDM is obtained by dividing the angle of the EI into equal intervals. Intensities in each subdivision are isotropic. On the other hand, intensity distribution for MCDM is obtained by dividing the angle of the EI according to the number of Gaussian quadrature points under consideration (Fig. 1b). In Figs. 1a and 1b, for both the cases, angular thickness of the DP and location of EI in each DP are shown for five EIs.

Note from Figs. 1a and 1b that, unlike CDM, the distribution of EI is uneven in the case of MCDM, and the weighted mean of EI at corresponding Gaussian points is considered for calculation. The distribution of EI in MCDM is analytically better represented than CDM.

In CDM, EI I in a DP, with angular thickness $\mathrm{d}\alpha$, is defined as

$$I(\alpha) = \frac{\delta \dot{Q}}{\mathrm{d}A \sin\alpha \, \mathrm{d}\alpha} \tag{1}$$

where $\delta\dot{Q}$ is the differential energy flow rate through the elemental area $\mathrm{d}A$ and α is the planar angle of EI measured from the surface under consideration.

In a general enclosure, heat flux at the control volume central surface point (CVSCP), m, due to a semicircle of EIs focusing at point m in the direction M is given by

$$q_{M,m} = \int_{\alpha=0}^{\pi} I(\alpha)\sin\alpha \, \mathrm{d}\alpha \tag{2}$$

To get the net heat flux at the same point, intensities in the other semicircle ($\pi < \alpha \leq 2\pi$) has to be integrated. In Eq. (2), EI is found using the integral form of the radiative transfer equation given by

$$I[\tau(\alpha)] = I(0)\exp[-\tau(\alpha)\eta] + \int_{\tau'=0}^{\tau(\alpha)} S(\tau')\exp[-(\tau(\alpha)-\tau')\eta] \, \mathrm{d}(\tau'\eta) \tag{3}$$

where $\tau(\alpha)$ is the optical thickness at any point along a particular effective ray path direction α and η is the OTC.[5] In Eq. (3), the source function $S(\tau)$ is given by

$$S(\tau) = (1-\omega)I_b + \frac{\omega}{2\pi}\int_{\alpha'=2\pi} I[\tau(\alpha')] \, p(\alpha' \to \alpha) \, \mathrm{d}\alpha' \tag{4}$$

where $p(\alpha' \to \alpha)$ is the phase function. In case of linear anisotropic scattering, it is given by

$$p(\alpha' \to \alpha) = 1 + a_1 \sin\alpha \sin\alpha' \tag{5}$$

From Eq. (4), the source function in terms of incident radiation G and heat flux q can be written as

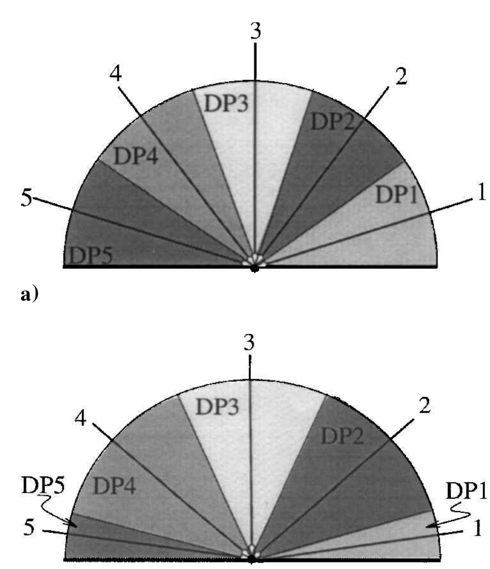

Fig. 1 Distribution of effective intensities and angular thickness of the discrete planes in a) CDM and b) MCDM.

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$$S(\tau) = (1-\omega)I_b + (1/2\pi)[\omega G + (a_1\omega)\sin\alpha\, q] \tag{6}$$

Under radiative equilibrium condition, $\nabla \cdot q = 0$, and this yields $G = 2\pi I_b$. In this situation, the source function simplifies to

$$S(\tau) = (1/2\pi)[G + (a_1\omega)\sin\alpha\, q] \tag{7}$$

In the finite difference procedure, to find EI at any point m, integration in Eq. (3) is performed over a short path leg. Over this path leg, the source function given by Eq. (6) can be assumed constant. In this situation, Eq. (3) takes the form

$$I[\tau(\alpha)] \approx I(0)\exp[-\tau(\alpha)\eta] + S[\tau(\alpha)/2][1-\exp(-[\tau(\alpha)\eta] \tag{8}$$

where $S[\tau(\alpha)/2]$ is the value of the source function at the middle of the integration path. In Eq. (8), $I(0)$ is the boundary intensity. For a diffuse gray boundary wall with emissivity ϵ_w and temperature T_w, the boundary intensity is given by

$$I(0) = \epsilon_w \frac{\sigma T_w^4}{2} + \frac{1-\epsilon_w}{2}\int_{\alpha=0}^{\pi} I(\alpha)\sin\alpha\, d\alpha \tag{9}$$

where the first term on the right-hand side represents the emitted component of EI and the second term is the reflected component of the EI. In CDM, the heat flux $q_{M,m}$ at any CVSCP m can be calculated by the numerical integration of Eq. (2) by considering the finite number of EIs $I(\alpha)$ incident on the CVSCP and contained within the semicircle corresponding to the directions M. Thus, Eq. (2) can be written as

$$q_{M,m} = \left[\int_0^{\pi} I(\alpha)\sin\alpha\, d\alpha\right]_{M,m} = \left[\sum_{n=1}^{N} c_n I(\alpha_n)\right]_{M,m} \tag{10}$$

where in general

$$c_n = |\cos[\alpha_n + (\Delta\alpha_n/2)] - \cos[\alpha_n - (\Delta\alpha_n/2)]| \tag{11}$$

and || indicates the absolute value. The term $\Delta\alpha_n$ is the angle over which the nth EI, $I(\alpha_n)$, is acting. The effective incident radiation G at any CVSCP m is found from the relation

$$G_m = \left[\int_0^{2\pi} I(\alpha)\, d\alpha\right]_m = \left[\sum_{n=1}^{2N} I(\alpha_n)\Delta\alpha_n\right]_m \tag{12}$$

where, in the preceding equations, N is the number of EIs over $0 \le \alpha \le \pi$.

Heat flux $q_{M,m}$ and the incident radiation G_m at the CVSCP m given by Eqs. (9) and (11) are expressed in MCDM using Gaussian quadrature (Fig. 1b), respectively, in the following form:

$$q_{M,m} = (\pi/2)\Sigma wf(\mu)I(\mu)\sin\alpha \tag{13}$$

$$G_m = \tfrac{1}{2}\Sigma wf(\mu)I(\mu) \tag{14}$$

where $\mu = \cos\alpha$ and $wf(\mu)$ is the weight factor.

For the purpose of calculation, quantities such as EI I, source function S, boundary intensity $I(0)$, heat flux $q_{M,m}$, and effective incident radiation G_m in the preceding equations are nondimensionalized as

$$I^* = \frac{I}{\sigma T_R^4/2}, \qquad S^* = \frac{S}{\sigma T_R^4/2}, \qquad I^*(0) = \left[\frac{I(0)}{\sigma T_R^4/2}\right]$$

$$\Psi_{M,m} = \frac{q_{M,m}}{\sigma T_R^4}, \qquad G_m^* = \frac{G_m}{\sigma T_R^4/2} \tag{15}$$

where T_R is the reference temperature and it has been chosen depending on the nature of the problem.

Results and Discussion

Improvements in MCDM over CDM are tested by solving some sample radiative transfer problems in nonradiative and radiative equilibrium situations. For this purpose, a two-dimensional square and one-dimensional Cartesian enclosure are chosen for radiative nonequilibrium and radiative equilibrium situations, respectively.

Nonradiative Equilibrium

In a nonradiative equilibrium situation, bounding walls of the two-dimensional square enclosure are considered black. The contained gray-homogeneous medium is absorbing–emitting. The boundary walls of the enclosure are cold, and the medium is isothermal with temperature T_g. In this situation, variations of nondimensional wall heat flux Ψ with normalized distance x/L_X along one of the walls (for example, the north wall) are studied for different value of extinction coefficients β. For a chosen number of rays, both CDM and MCDM results are compared with results from the DOM provided in Ref. 8. Here, for nondimensionalization of heat flux, medium temperature T_g has been chosen as reference temperature T_R.

In Fig. 2, variations of nondimensional heat flux Ψ along one of the walls of a square enclosure are presented. These results are presented for three values of extinction coefficient β, that is, 0.1, 1, and 5. For (20 × 1) control volume, both MCDM and CDM results have been obtained with 16 and 32 EIs, respectively. In Fig. 2, both MCDM and CDM results are compared with results from the DOM.[8] The reported DOM results are for (21 × 21) spatial with S_8 approximations (80 angular discretizations). From results presented in Fig. 2, note that MCDM with 16 EIs compares very well with 32 EIs of CDM and DOM with 80 angular discretizations.

Radiative Equilibrium

In the radiative equilibrium situation, for the one-dimensional black Cartesian encolsure, the south wall is at a finite temperature T_S, whereas the north wall is at zero temperature, $T_N = 0$. The gray and homogeneous medium is absorbing, emitting, and anisotropically scattering. In this case, variations of wall heat flux Ψ with enclosure optical thickness τ_L from MCDM and CDM are compared with the exact results.[5] Here, south wall temperature T_S has been taken as the reference temperature.

Table 1 Absolute % error in wall heat flux Ψ for MCDM and CDM with respect to exact results; $a_1\omega = +1.0$, 6 EIs, and 50 control volumes

τ_L	Exact	MCDM	Error	CDM	Error
0.5	0.7716	0.7632	1.0886	0.7622	1.2182
1.0	0.6409	0.6401	0.1248	0.6330	1.2326
2.0	0.4860	0.4831	0.5967	0.4823	0.7613
3.0	0.3883	0.3898	0.3863	0.3956	1.8800
5.0	0.2796	0.2804	0.2861	0.2957	5.7582

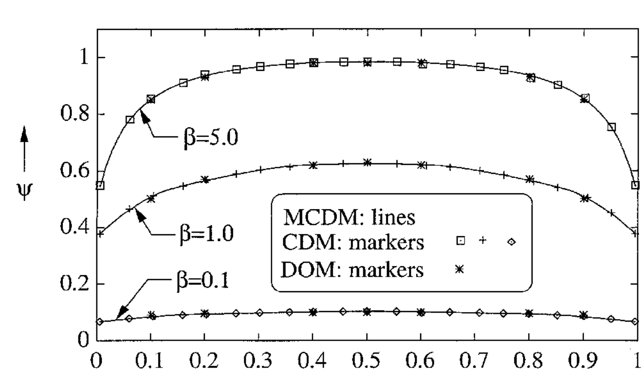

Fig. 2 Variation of nondimensional wall heat flux Ψ along one of the walls of a two-dimensional square enclosure; comparison of CDM and MCDM results with DOM results.

Table 2 Absolute % error in wall heat flux Ψ for MCDM and CDM with respect to exact results; $a_1\omega = -1.0$, 6 and 16 EIs, and 50 control volumes

		6 EIs				16 EIs			
τ_L	Exact	MCDM	Error	CDM	Error	MCDM	Error	CDM	Error
0.5	0.6467	0.6392	1.1597	0.6438	0.4484	0.6451	0.2474	0.6487	0.3093
1.0	0.4849	0.4852	0.0619	0.4833	0.3300	0.4873	0.4949	0.4907	0.5800
2.0	0.3267	0.3308	1.2550	0.3275	0.2449	0.3294	0.8264	0.3345	2.3875
3.0	0.2441	0.2524	3.4002	0.2498	2.3351	0.2502	2.4990	0.2556	4.7112
5.0	0.1623	0.1704	4.9908	0.1723	6.1643	0.1680	3.5120	0.1766	8.8108

For the radiative transfer problem in the one-dimensional Cartesian enclosure, for the problem under radiative equilibrium, comparisons of wall heat Ψ results of MCDM and CDM with that of exact results are made in Tables 1 and 2. In Tables 1 and 2, comparisons are made for $\tau_L = 0.5$, 1, 2, 3, and 5. For results presented in Table 1, anisotropy $a_1\omega$ is taken as $+1$, whereas that for the results presented in Table 2, $a_1\omega = -1$. For linear anisotropic phase function considered in this work, these two values represent 100% forward and 100% backward scattering situations, respectively. For results presented in Table 1, comparisons are made with 6 EIs and 50 control volumes. In Table 2, for 50 control volumes, 6 and 16 EIs are used in MCDM and CDM respectively. Note from Tables 1 and 2 that for equal numbers of EIs and control volumes, accuracy in MCDM is greater than that of CDM.

Note that for the absorbing–emitting isothermal medium under nonradiative equilibrium situation considered in this work, the source function in Eq. (6) is unity. Hence, to arrive at Eq. (8), no approximation is required. Thus, for calculation of wall heat flux, effective intensities have been found in a single step, starting from the point of origin to the point of interest. In this situation, the number of control volumes chosen is just to get the radiative information at the locations. However, in situations in which source function is not constant, for example, even the absorbing–emitting case of radiative equilibrium situation considered in this work, to get Eq. (6) from Eq. (8) the integration path leg has to be small enough to comply with the assumption that source function remains constant over the integration path. This justifies the consideration of the large number of control volumes in the calculation of Ψ in Tables 1 and 2.

Conclusions

CDM has been modified for the solution of radiative transfer problems with participating medium. This has been done by utilizing the concept of Gaussian quadrature to determine the angular locations of the effective intensity and angular thickness of the DPs. The present method for discretization and angular integration gives the positive features of the DOM. The improvements in MCDM have been tested with a two-dimensional square and one-dimensional gray Cartesian enclosure for nonradiative and radiative equilibrium situations, respectively. For an equal number of rays, MCDM results for heat flux have been found to be more accurate than CDM results.

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